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The goal of this program has been the formulation of the mathematical foundations for a theory of linear systems defined in Hilbert Space. Since the classical Hilbert space structure does not include a time concept the theory has been formulated in a resolution space setting, a Hilbert space to which a time concept has been axiomatically adjoined. The work has included a study of the underlying mathematics of resolution space, applications to stability theory, optimal stochastic systems design, and multidimensional systems.

In the "mathematics area" we have developed a Fourier representation theory for systems defined in a uniform resolution space and formulated the concept of a reproducing kernel resolution space. The latter study is intimately related to the spectral factorization problem and has included an investigation of the fundamental existence criteria and a representation theory for this tool which underlies much of linear system theory.

The stability work has been centered around the formulation of a generalized Nyquist criterion. The major result here is an extremely powerful sufficient condition for the stability of a feedback system which is applicable to nonlinear as well as linear feedback systems and subsumes much of the classical literature. Indeed, the result is known to be necessary in a number of cases and we conjecture that it is a necessary and sufficient condition for stability in a large class of feedback systems.

Our results on optimal stochastic systems are predicated on the formulation and solution of a "basic stochastic optimization" problem. By applying the explicit solution to this problem in a variety of cases explicit solutions are obtained to a number of control and estimation problems. The resultant solutions

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are formally similar to the classical frequency domain solutions to these problems, but by virtue of the resolution space setting employed, they apply to time-varying and distributed systems.

A final area of activity in which our abstract theory has been specialized to a more applied setting has been our study of multidimensional systems. The major result here has been the formulation of a multidimensional Nyquist criterion for such systems. Indeed, not only does one obtain a necessary and sufficient condition for stability, but moreover this is achieved through a one dimensional Nyquist polt rather than the n-dimensional polt which might be expected.

## Publications Completed under AFOSR Grant 74-2631

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DeSantis, R. M., Saeks, R., and L. J. Tung, "A Basic Stochastic Optimization Problem", March 1978.

Saeks, R., DeCarlo, R., Tung, L. J., DeSantis, R. M. and D. Brandon, "Resolution Space, Networks, and Non-Self-Adjoint Spectral Theory, II, (in press).

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Sain, M. K., "Applications of Modern Algebra in Engineering: Introductory Lecture Notes," Res. Rpt., Texas Tech Univ., May, 1975.

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## Conference Participation under AFOSR Grant 74-2631

Saeks, R., Attendance at the 1974 IEEE Conf. on Decision and Control, Phoenix, Nov., 1975, (paper was presented)

Saeks, R., and R. A. DeCarlo, Attendance at the 17th Midwest Symp. on Circuits and Systems, Lawrence, Sept. 1974, (a paper was presented by Mr. DeCarlo).

Saeks, R., Attendance at the IEEE Inter. Symp. on Circuits and Systems, San Francisco, April 1974.

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Saeks, R., and R. A. DeCarlo, Attendance at the 18th Midwest Symp. on Circuits and Systems, Montreal, Aug 1975 (papers were presented by Prof. Saeks and Mr. Decarlo)

Saeks, R., Attendance at the IEEE Inter. Symp. on Circuits and Systems, Boston, April 1975.

Strauss, M., Attendance at the 80th AMS National Meeting, San Francisco, Jan. 1975 (a paper was presented).

Saeks, R. Attendance at the 1976 IEEE Inter. Symp. on Circuits and Systems, Munich, April 1976, (a paper was presented).

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Saeks, R., DeCarlo, R. A., Murray, J., and L. Tung, Attendance at the 2nd Int. Symp. on the Operator Theory of Networks and Systems, Lubbock, Texas, Aug. 1977, (a paper was presented by Dr. Tung).

## Invited Lectures at Universities Related to AFOSR Grant 74-2631

Saeks, R., Rice Univ., Nov. 1973.

Saeks, R., Univ. of Notre Dame, April, 1976.

Saeks, R., Univ. of Warwick, May, 1976.

Saeks, R., Univ. of Pennsylvania, Jan. 1977.

Saeks, R., Univ of Montreal, McGill Univ., and Concordia Univ., (Joint Seminar), Nov. 1977.

Saeks, R., Purdue Univ., Dec. 1977.

Saeks, R., Univ of Calif., at Berkeley, Jan 1978.

Saeks, R., Univ. of Calif., at Los Angeles, April 1978.

## Professional Activities Related to AFOSR Grant 74-2631

Saeks, R., Co-Chairman of the 2nd Inter. Symp. on the Operator Theory of Networks and Systems, Texas Tech Univ., Aug. 1977.

Saeks, R., Co-Chairman of the 20th Midwest Symp. on Circuits and Systems, Texas Tech Univ., Aug. 1977.

Saeks, R., Member of the Editorial Board for the Special Issue of the IEEE Proc. on "New Vistas in System Theory," Jan. 1976.

Saeks, R., Member of the Editorial Board for the Special Issue of the J. of the Franklin Inst., on "New Vistas in System Theory," Jan 1976.

Saeks, R., Member of the Organizing Committee for the Inter. Symp. on the Operator Theory of Networks and Systems, Montreal, Aug. 1975.

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DeCarlo, R. A. Ph.D. Thesis, Texas Tech Univ., 1976.

Tung, L. Ph.D. Thesis, Texas Tech Univ., 1977

Brandon, D., M.S. Thesis, Texas Tech Univ., 1977.

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The results of a five year study directed at the formulation of mathematical foundations for a theory of linear systems defined in Hilbert space are summarized. Since the classical Hilbert space structure does not include a time concept, the theory has been formulated in a resolution space setting, a Hilbert space, to which a time concept has been axiomatically adjoined. The work has included a study of the underlying mathematics of resolution space, applications to stability theory, optimal stochastic systems design, and multidimensional systems.

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